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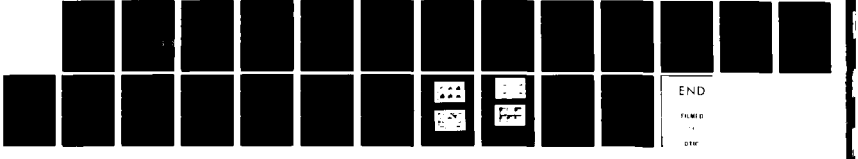

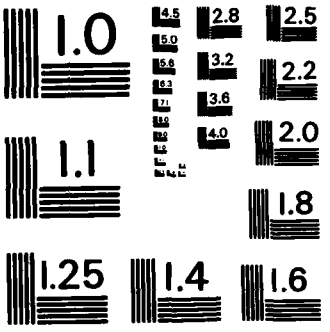

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It is shown how the ordinal image segmentation problem fits into an earlier well-devloped model for hierarchical clustering. Certain techniques suggested by this model are investigated and implemented on real data. The results are compared to those achieved by segmentation techniques that involve region mergers based on various notions of scatter. The techniques are examined from both an order theoretic and statitical viewpoint.

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A CLUSTER ANALYSIS PROGRAM FOR IMAGE SEGMENTATION

by

M. F. Janowitz
Department of Mathematics and Statistics
University of Massachusetts
Amherst, Ma 01003

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A Cluster Analysis Program For Image Segmentation

M. F. Janowitz *

I. Introduction.

A more detailed description of the contents of this paper occurs in [7] and [8]. The present version is intended only to be a survey, and for that reason will omit proofs of results. The thrust of the work is to relate the image segmentation problem to a rather general model for cluster analysis that was introduced in [3]. Some techniques suggested by the model are described, and their implementation illustrated. Section 2 is devoted to some background material from the theory of partially ordered sets, with Section 3 containing the image segmentation model. Section 4 has an elementary discussion of some of the underlying statistical considerations, with Sections 5 and 6 devoted to the description of certain segmentation techniques suggested by the model. Finally, in Section 7 the techniques are implemented on real data, and the results are presented.

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II. Background material from the theory of partially ordered sets.

A basic familiarity with the theory of partially ordered sets and lattices will be assumed. Despite this, it will be convenient to specifically develop certain terminology here. Unless otherwise specified, all partial orders will be denoted $\leq$, with $\subseteq$ reserved for set inclusion. The symbol $\mathcal{P}(X)$ represents the Boolean algebra of all subsets of the set X, ordered by set inclusion. Unions and intersections have the standard symbols of $\cup$ and $\cap$, and it will be useful to let R denote the nonnegative real numbers, ordered in the usual manner.

Let P, Q be partially ordered sets. A mapping $\phi : P \to Q$ is called isotone if $p_1 \leq p_2$ in P implies that $\phi(p_1) \leq \phi(p_2)$ in Q. It is residuated if it is isotone and there is an isotone mapping $\phi^+ : Q \to P$ such that $p \leq \phi^+\phi(p)$ and $q \geq \phi\phi^+(q)$ for every $p \in P$, $q \in Q$. The mapping ϕ^+ is the residual mapping associated with ϕ, and it is completely determined by ϕ; likewise, ϕ is completely determined when ϕ^+ is specified, so for a given residual mapping θ, it will often be convenient to write θ^* for the residuated mapping with which θ is associated. The notation Res(P,Q) or Res(P) when P = Q will be used to denote the set of all residuated mappings from P into Q, with $\text{Res}^+(Q,P)$ and $\text{Res}^+(P)$ for the corresponding sets of residual mappings. It turns out that residuated and residual mappings play a natural role in the Jardine-Sibson model [9] for hierarchical clustering. This is because ([3], Lemma 4.1, p.60) $C: R \to \mathcal{P}(X)$, where X is a finite nonempty set, is residual if and only if it satisfies the following three conditions:

(1) C is isotone.

(2) $C(h) = X$ for some $h \in R$.

(3) Corresponding to each $h \in R$. there is a positive real number $\delta = \delta(h)$ such that $C(h) = C(h + \delta)$.

If X is the set of all 2 element subsets of the set P, this turns out to be equivalent to C being a numerically stratified clustering in the sense of Jardine and Sibson [9], p.61.

The theory of residuated mappings is rather extensively developed in [2], and the reader is referred to that source for further information.

III. The image segmentation problem.

The underlying input data is a function $F: P \to R$, where P represents a closed bounded rectangle. One thinks of P as representing a picture having certain natural regions that are characterized in some known manner by the values of F. This might involve regions on which F is constant, or some sort of texture measure, or some combination of attributes of F. Unfortunately, no direct knowledge of the values of F is available. Rather, the actual input is a function $G: X \to R$, where $X = \{1,2,...m\} \times \{1,2,...,n\}$ for suitable positive integers m and n. One can think of G as being some sort of sample of F or as somehow summarizing the values of F, possibly by averaging F over some small subregions of P. Often there is some noise involved in the passage from F to G, so G can only be regarded as an estimate of F. The idea is to use G to somehow recapture the principal regions of P as they are defined by F. A more precise and detailed

description of these underlying assumptions occurs in [7], but the present version is sufficient for the current discussion.

A rather broad model for hierarchical clustering was presented in [3]. It included both the Jardine-Sibson model and a graph-theoretic model due to Matula [10]. It will now be shown that it also includes the image segmentation problem as we have stated it. Recall that the actual input data is a mapping $G: X \to R$, where $X = \{1,2,\ldots,m\} \times \{1,2,\ldots,n\}$. The idea is to transform G into a function $H: X \to R$, where H has fewer distinct levels of range values than does G. The values of H should be thought of as an estimate of the regions represented by F. There is a natural bijection between mappings $G: X \to R$ and elements of $\mathrm{Res}^+(R, \mathcal{Q}(X))$. It is given by associating with G the unique residuated mapping $G^*: \mathcal{Q}(X) \to R$ to which it extends, the extension being given by the rule $G^*(M) = \bigvee\{G(m) : m \in M\}$. One then takes the residual mapping G^+ associated with G^*. Thus the image segmentation problem may be viewed as the study of transformations of $\mathrm{Res}^+(R, \mathcal{Q}(X))$ into $\mathrm{Res}^+(R, \mathcal{Q}(Y))$, where $Y \subseteq X$. Such transformations will be called segmentation methods. The reason for $Y \subseteq X$ will be apparent from the concrete examples that will be presented. This places the segmentation problem squarely within the framework of the model contained in [3].

No assumption has thus far been made as to whether the input data has ordinal or numerical significance. Suppose that the input data has only ordinal significance in that the actual numerical values of G have no significance, but one might still be able to attach some significance to an assertion of the form $G(i,j) < G(k,l)$. Is it possible to describe the class of segmentation methods that are suitable for this type of data? Indeed it is, but before the

description can be presented, some additional terminology is required. Suppose one is given a segmentation method and a residual mapping θ on R. To say that $\mathfrak{Z}$ is θ-compatable ([3], p.68) is to say that for every $C \in Res^{+}(R,\mathcal{Q}(X))$, it is true that $\mathfrak{Z}(C\bullet\theta) = \mathfrak{Z}(C)\bullet\theta$. A minimal requirement for $\mathfrak{Z}$ is that it be θ-compatable for all order automorphisms θ of R. Such techniques are called monotone equivariant by Jardine and Sibson. A precise mathematical proof is given in [5] of the fact that every segmentation technique suitable for use with ordinal data is in a sense equivalent to a monotone equivariant technique. The equivalence is the fact that if the outputs were rank ordered, they would be identical. This now makes one inquire into the nature of monotone equivariant techniques. This question was posed and answered in [4] (Theorem 1, p.149). For $C \in Res\ (R,\mathcal{Q}(X))$, one says that h ($h \in R$) is a splitting level of C in case h is the image of some subset of X under C^{*}, the residuated mapping with which C is associated. If $0 < h_1 < \ldots < h_t$ denotes the sequence of splitting levels of C, then the segmentation method $\mathfrak{Z}$ is monotone equivariant if and only if it is true that:

(i) every splitting level of $\mathfrak{Z}(C)$ is a splitting level of C,

(ii) the sequence $\mathfrak{Z}C(0) \leq \mathfrak{Z}C(h_1) \leq \ldots \leq \mathfrak{Z}C(h_t) = Y$ depends only upon the sequence $C(0) < C(h_1) < \ldots < C(h_t) = X$ and is independent of the actual values of the h_i's;

(iii) conditions (i) and (ii) hold for every C in $Res^{+}(R,\mathcal{Q}(X))$.

One can associate with each subset T of $Res^{+}(R)$ the collection $\alpha(T)$ of all segmentation methods that are θ-compatable for every $\theta \in T$. If the nature of the input data makes compatability with a certain set T of residual mappings desirable, then the determination of $\alpha(T)$ will produce a description of the most general type of segmentation technique that one should properly use. We have just noted that if T denotes the set of all order automorphisms of R, then $\alpha(T)$ is the set of all monotone equivariant mappings. An extremely important result is provided by F. Baulieu [1]. It yields an explicit description of all classes $\alpha(T)$ that can arise for T containing the set of all order automorphisms of R. Here is a description of these classes:

(S1) Flat methods. $\exists C(h)$ depends only on C(h).

(S2) Semiflat methods. $\exists C(h)$ depends upon C(h) and C(0).

(S3) Divisive methods. $\exists C(h_i)$ depends upon $C(h_i)$, $C(h_{i+1})$, ..., $C(h_t)$, where h_t is the highest splitting level of C.

(S4) SF(1) methods. $\exists C(0)$ depends only on C(0); for $h_i \neq 0$, $C(h_i)$ depends upon $C(h_i)$, $C(h_{i+1})$, ..., $C(h_t)$.

(S5) Monotone equivariant methods.

The simplest of these classes of segmentation methods is of course the class of flat methods, and it is to this class that we now direct our attention. Such methods are easy to describe and easy to implement. Given a fixed non-

empty finite set X and an increasing sequence

$$M_1 \subset M_2 \subset \ldots \subset M_t = X$$

of subsets of X, where $M_i = C(h_i)$, the goal is to produce a sequence

$$N_1 \subseteq N_2 \subseteq \ldots \subseteq N_t = Y$$

of subsets of X that somehow summarize or represent the underlying picture that led to the original input data. In that N_i depends only upon M_i, this amounts to defining an isotone mapping γ on $\mathcal{P}(X)$.

Not every isotone mapping on $\mathcal{P}(X)$ is appropriate for defining a flat segmentation method. One wants to attach some spatial significance to the decision as to whether a point x belongs to $\gamma(M)$. One way of doing this is to insist that γ be <u>point-based</u> in that for each $x \in \gamma(X)$ there is a subset $N(x)$ of X containing x such that:

(PB1) $\gamma(\emptyset) = \emptyset$, and $\gamma(N(x)) \neq \emptyset$.

(PB2) For $A \subseteq N(x)$, if $\gamma(A) \neq \emptyset$, then $x \in \gamma(A)$.

(PB3) For $M \subseteq X$, $x \in \gamma(M)$ if and only if $x \in \gamma(M \cap N(x))$.

It is immediate that for every subset M of X, $\gamma(M) = \bigcup_{x \in \gamma(X)} \gamma(M \cap N(x))$. If one wants the output of γ to be independent of the polarity of the image, it is useful to also insist that γ <u>preserve complements</u> in that it also satisfy:

(PB4) For $M \subseteq X$, $\gamma(X \setminus M) = \gamma(X) \setminus \gamma(M)$.

The local version of this is the content of Theorem 1.

Theorem 1. For a point-based isotone mapping γ on $\mathcal{P}(X)$, axiom (PB4) is equivalent to the assertion that for each subset A of N(x), exactly one of $\gamma(A)$ and $\gamma(N(x) \setminus A)$ shall be nonempty.

If $\{N(x)\}$ and $\{M(x)\}$ are each families of neighborhoods satisfying axioms (PB1) through (PB3), then so is $\{M(x) \cap N(x)\}$. In view of this there is no harm in assuming;

(PB5) If M(x) satisfies (PB1) through (PB3), then $N(x) \subseteq M(x)$.

Such a minimal family of subsets of X will be called the system of neighborhoods associated with γ. Henceforth, when we speak of a point-based isotone mapping γ, it will always be assumed that $\{N(x)\}_{x \in \gamma(X)}$ denotes the system of neighborhoods associated with γ.

Definition. The point-based isotone mapping γ is said to be frequency-defined if there exist positive integers j and k such that for every $x \in \gamma(X)$, (i) $k = \#N(x)$, and (ii) $x \in \gamma(M)$ if and only if $j \leq \#(M \cap N(x))$. Here #A denotes the cardinality of the set A.

Theorem 2. Let γ be a point-based, complement-preserving isotone mapping on $\mathcal{P}(X)$. Necessary and sufficient conditions for γ to be frequency-defined are that:

(i) the neighborhoods N(x) all have the same cardinality k, where k is odd, and

(ii) if A_x has minimal cardinality among those subsets

A of N(x) for which $x \in \gamma(A)$, then

$\#A_x = (1 + k)/2$.

To say that the mapping γ is a join homomorphism is to say that $\gamma(M \cup N) = \gamma(M) \cup \gamma(N)$. There is a dual notion of meet homomorphism and to say that γ is a homomorphism is to say that it is both a join and a meet homomorphism. The next theorem shows that these conditions are rather powerful, and will only be met in trivial situations.

Theorem 3. Let γ be a point-based isotone mapping on $\mathcal{P}(X)$. Then (1a) $\Leftrightarrow$ (1b), (2a) $\Leftrightarrow$ (2b) and (3a) $\Leftrightarrow$ (3b).

(1a) γ is a join homomorphism.

(1b) For each $x \in \gamma(X)$ there is a subset A_x of N(x) such that $x \in \gamma(M)$ if and only if $M \cap A_x \neq \emptyset$.

(2a) γ is a meet homomorphism.

(2b) Corresponding to each $x \in \gamma(X)$ there is a subset B_x of N(x) such that $x \in \gamma(M)$ if and only if $B_x \subseteq M$.

(3a) γ is a homomorphism.

(3b) For each $x \in \gamma(X)$ there corresponds an element y_x of N(x) such that $x \in \gamma(M)$ if and only if $y_x \in M$.

Corollary 4. If γ is also complement-preserving, then all six conditions of the theorem are mutually equivalent.

A translation on X is a mapping of the form $T_{p,q}$ where p,q are fixed integers and $T_{p,q}(i,j) = (p+i,q+j)$. Unless $p = 0 = q$, the domain and image of $T_{p,q}$ will be proper subsets of X. We agree to call the point-based isotone mapping translation-invariant provided it satisfies:

(PB6) If N(x) is contained in the domain of the translation T, and if $y = T(x)$, then $N(y) = T(N(x))$, and for $A \subseteq N(x)$, $x \in \gamma(A)$ if and only if $y \in \gamma(T(A))$.

Remark 5. If M is acted upon by a translation to produce N, it is desirable that $\gamma(M)$ produce $\gamma(N)$ when it is acted on by that same translation. As long as γ is translation-invariant, $M \subseteq$ domain T, and $M \cup T(M) \subseteq \gamma(X)$, it is easy to see that this is true. An example is provided in [8] to show that this can fail even if γ is translation-invariant.

Remark 6. The actual construction of a point-based isotone mapping on $\mathcal{P}(X)$ may now easily be understood. A system of neighborhoods $\{N(x)\}$ for points x in some subset Y of X is first chosen. One then defines a family of mappings $(\gamma_x)_{x \in Y}$, where γ_x is an isotone mapping on N(x) such that:

(1) $\gamma_x(\emptyset) = \emptyset$, and $\gamma_x(N(x)) = \{x\}$.

To produce a complement-preserving mapping, one also wants

(2) For $A \subseteq N(x)$, exactly one of $\gamma_x(A)$ and $\gamma_x(N(x) \setminus A)$ is nonempty.

The mapping γ is now defined by the rule $x \in \gamma(M)$ if and only if $\gamma_x(M \cap N(x)) = \{x\}$. This is the technique that will be used for the remainder of the paper.

IV. Underlying statistical considerations.

The construction of a flat segmentation method involves the definition of an isotone mapping γ on $\mathcal{P}(X)$. The input data represents a subset M of X that is an estimate of the subset M^* of X that is the true input data. The idea is to try to define γ so that $\gamma(M)$ is in some sense a better estimate of M^* than is M. A crude statistical model may be constructed by assuming that membership in M has probability p ($p > 0.5$) of providing a correct estimate of membership in M^*, that membership in $X \setminus M^*$ has a probability q ($q > 0.5$) of providing a correct estimate of membership in $X \setminus M^*$, and that these probabilities are independently distributed over the members of X. An interior point of M^* is defined to be a point x for which $N(x) \subseteq M^*$; dually, x is an exterior point of M^* if $N(x) \subseteq X \setminus M^*$. The gain of the isotone mapping γ is defined to be the sum of the probability of correct classification for an interior point of M^* and that of an exterior point. Needless to say, these are idealized concepts, as M^* is precisely the unknown set that we are trying to estimate. If $G(\gamma)$ denotes the gain of γ, we then have

Lemma 7. Let γ be complement-preserving with $k = \#N(x)$ odd, and let γ' be the frequency-defined, point-based isotone mapping having the same neighborhood system as γ. Then $G(\gamma) \leq G(\gamma')$.

Theorem 8. If $p = q$, and if γ and γ' have the same system N(x) of neighborhoods with γ' frequency-defined, then $G(\gamma) \leq G(\gamma')$.

When p = q, the above theorem shows that the "gain" from a flat segmentation method can be maximized by using a point-based isotone mapping that is complement-preserving and frequency-defined. With this thought in mind, we shall concentrate on such mappings, defining them using k by k neighborhoods of points with k an odd integer. The j/k^2 rule will be the unique such mapping defined on a k by k neighborhood, and the 3/5 rule will refer to the γ that is based on a point together with its 4 immediate direct neighbors (North, South, East and West). These rules were discussed in some detail in [7] and [8], and examples were given there to show that near the boundary of M^*, the use of these rules can actually decrease the probability of correct classification. This suggests using (see [8], Table 3) an isotone mapping γ that is based on a weighted mean or on $M \cap N(x)$ containing certain desirable subsets. For example, one would be more likely to conclude that $x \in \gamma(A)$ if A were the left-hand subset than if A were the right-hand set:

0	0	0	0	0		1	0	0	1	0
0	1	1	1	0		0	0	0	0	1
0	1	1	1	0		1	0	0	1	0
0	1	1	1	0		0	1	0	0	1
0	0	0	0	0		1	0	0	0	1

V. The SEGMENT Program.

This was described in some detail in [6], [7] and to a lesser extent in [8]. For the reader's convenience, a brief description is also included here.

A. Input. An m by n matrix A having nonnegative integers as entries.

B. Prefiltering. A k by k mean or median filter is applied to smooth the data of A, and the output is rounded to produce B. Spot noise is removed by deleting the highest and lowest i values when computing the k by k mean.

C. Thinout. A frequency count is made of the values appearing in B. Those values that occur with frequency less than some threshhold are deleted, and a nearest value rule used to reassign them. Alternately, the k highest occuring values can be retained. The resulting matrix is denoted C.

D. Suppose matrix C has data values $j_1, j_2, \ldots, j_k$. The nondirected 3 by 3 dispersion of value j_1 is defined as follows: For each point in C having value j_1, look at a 3 by 3 neighborhood centered on that point. Calculate the number of points in that neighborhood that do not have value j_1, and average this figure over all points having value j_1. This is the dispersion for cluster j_1. One can do a similar calculation for $j_2, \ldots, j_k$. An analogous definition can be formulated for larger neighborhoods, and certain options are included that compute directional versions of the dispersion.

E. Various options exist for deleting one or more of the data values having high dispersion levels. The simplest is to just delete the highest value dispersion. Other options might include deleting all data values whose dispersion exceeds 0.9 on the first pass, and then dropping this cutoff down by increments of 0.1 until a stopping criterion is reached. Various choices also exist for the reassignment of points whose value has been deleted. They proceed on both a global and a local basis. Cluster means

can be computed for each remaining cluster, and pixels reassigned to the cluster to which their input value is closest, or an entire cluster can be reassigned to the cluster to which its mean is closest. A second technique involves the reassignment of points to the next higher or lower cluster that is still valid, either on a local or a global basis. As these techniques have been described, they are not monotone equivariant, but if the data are first rank ordered, and the output is labeled according to which value corresponds to which rank, the resulting techniques do become monotone equivariant.

VI The SLICER program.

A crude segmentation program can easily be devised using the material of Sections III and IV. It has two phases:

Phase 1. Find levels at which to slice the data.

Phase 2. Make appropriate slices and use a j/k^2 rule to increase the probability of correct classification.

The Phase 2 portion of the program is a flat segmentation method. The program itself can either be used as a cleaning algorithm, or as a tool for obtaining a first approximation to the principal regions of a digital picture.

Description of Phase 1. A total of six methods of determining the slice levels are built into the program.

Method 1. Compute the mean M and standard deviation S of the input data, and take slices at M - 2S, M - S, M, M + S, and M + 2S. If the extreme values are outside the

data range, they are modified to bring them within the minimum and maximum values of the data.

Method 2. Slice at relative maxima of the histogram of the data.

Method 3. Slice at relative minima of the nondirected 3 by 3 dispersion.

Method 4. This is similar to Method 3, except that if A is the vector of distinct data values, then at level A[i], one is interested in those values that lie outside the range from A[i - 1] to A[i + 1].

Method 5. Slice at relative minima of the average variance in t by t neighborhoods.

Method 6. User inputs slice values.

Description of Phase 2. Having arrived at a list of slice levels, a j/k^2 rule is chosen. The actual slices are constructed so that they are halfway between the slice input levels. Thus if one wanted slices at 17, 20, 22,26, and 30, then one would look at the sets $M_1 \subset M_2 \subset M_3 \subset M_4 \subset M_5$, where $M_i = \{x: x < k_i\}$ with $k_1 = 18.5$, $k_2 = 21$, $k_3 = 24$, $k_4 = 28$ and k_5 = the maximum value of X. This forms 5 clusters, and they are assigned the values of 17, 20, 22, 26 and 30. There are also occasions when one would want the actual slice values to coincide with the input slice values. If γ represents the isotone mapping specified by the j/k^2 rule, the output would then be the sequence ;

$$\gamma(M_1) \subseteq \gamma(M_2) \subseteq \gamma(M_3) \subseteq \gamma(M_4) \subseteq \gamma(M_5).$$

If the actual slices are taken at levels k_1, k_2,...,k_t the implementation of this process is very simple. One starts by forming a Boolean matrix with 1 denoting a value $\leq k_1$ and 0 otherwise. One then looks at k by k regions. If there are j or more 1's in such a region, the output is 1; otherwise it is 0. The output for level k_2 is added to this, and the process continues until the supply of slice levels is exhausted.

The actual implementation of the SLICER program can be done more efficiently by first performing a k by k median filter, and then applying the algorithm described at the beginning of Phase 2. That way, if one wanted to examine the effect of various types of slicing, one would only have to perform the j/k^2 rule a single time, as this decision rule is in reality implemented by the median filter.

VII Some Examples.

Plates 1-4 contain a comparison of the output of the SEGMENT program with that of program SLICER. The plates all follow the same format, so a single explanation of them should suffice. There are 6 pictures per plate. The upper left picture is the original data set. The middle picture on the upper level is the SEGMENT output using a nondirected 3 by 3 dispersion, a 3 by 3 mean filter, and the version of the program that removes 1 cluster at a time, reassigning its members locally to the next higher or lower available cluster. The parameters were set to produce 5 clusters. The remaining pictures relate to the SLICER program, though it should be noted that the program was modified to make it iterate with a stopping criterion set for when an iteration did not change the cluster means. At each stage, cluster means were computed, and these values used as slice level input for the next iteration. The upper

right picture was the result of the first, third and fifth levels of option 1 of SLICER, the bottom left was the result of option 1, the bottom middle the result of 3 slices equally placed between the minimum and maximum values of tne input data, and the bottom right the result of 5 slice levels equally placed. Thus for 3 levels, the slices are taken 1/4, 2/4 and 3/4 of the way from the minimum to the maximum data value; for 5 levels, the slices are 1/6, 2/6, 3/6, 4/6 and 5/6 of the way. In each case a 5/9 rule was applied to produce the final output. No further cleaning algorithm was applied. It should be noted that the various SLICER outputs produced reasonable crude segmentations of the data. Further work is clearly needed on the initial selection of slice levels. The outputs are especially useful when one resorts to line printer representations, as one can intelligently reduce a picture to a desired number of grey levels.

Plate 3 is worthy of note. It illustrates one of the pitfalls of the SEGMENT program. This program operates essentially by merging clusters; no provision is made for a subsequent splitting of a cluster, unless the dispersion criterion deems that cluster to be too noisy to be maintained. The vehicle in the center of the picture is too large because of some early mergers that were not undone. In the lower right-hand corner of Plate 4, a portion of the Gulf Stream is visible. In all but 1 of the outputs, this cluster was merged with warm coastal water. This was caused largely by the iterations using cluster means as outputs. The original iteration more correctly merged the Gulf Stream into the same cluster as the land masses, thus correctly identifying it as being much warmer than the coastal water.

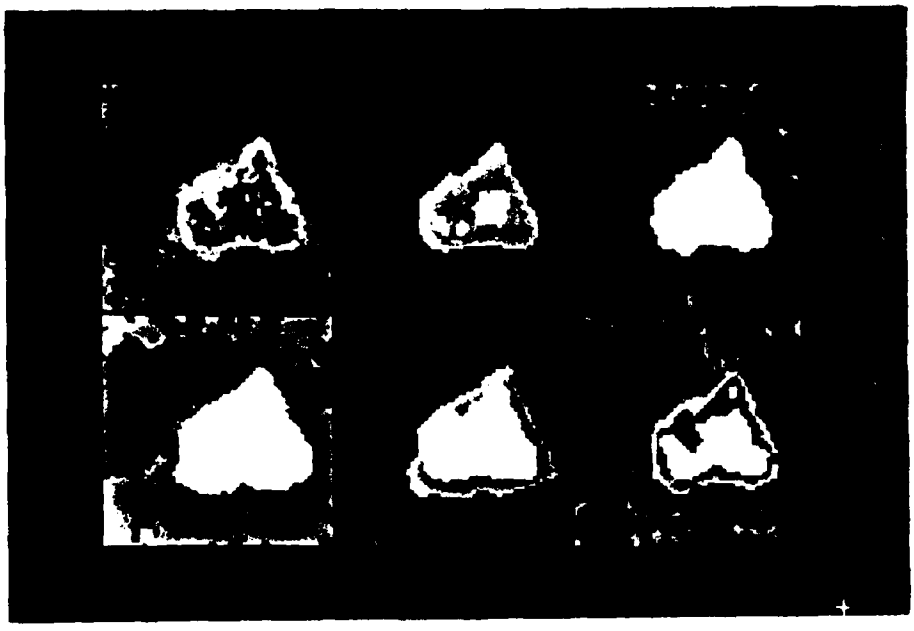

Plate 1. Portion of Record 21 of Westinghouse FLIR data tape.

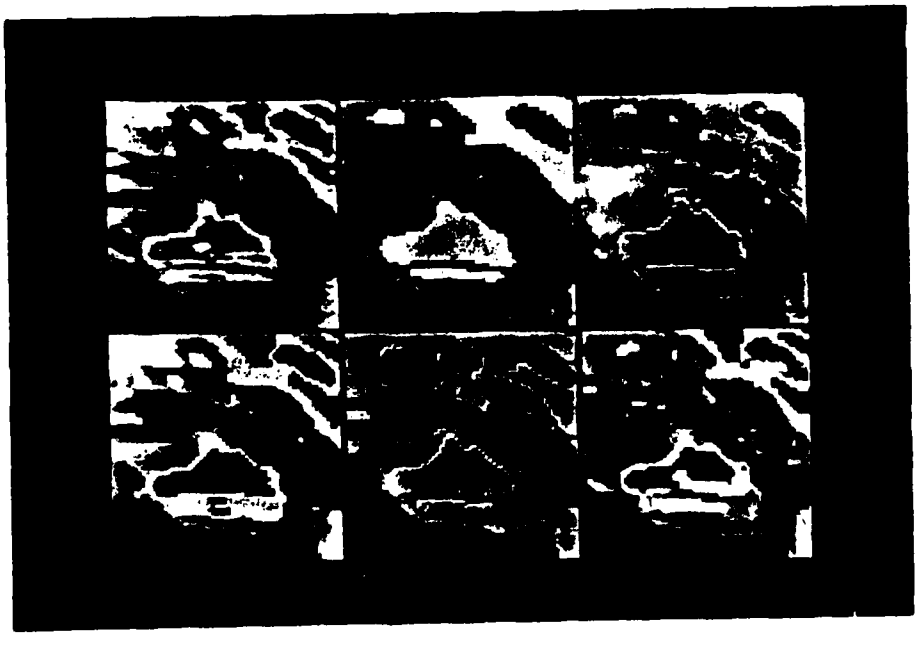

Plate 2. Portion of Record 33 of Westinghouse FLIR data tape.

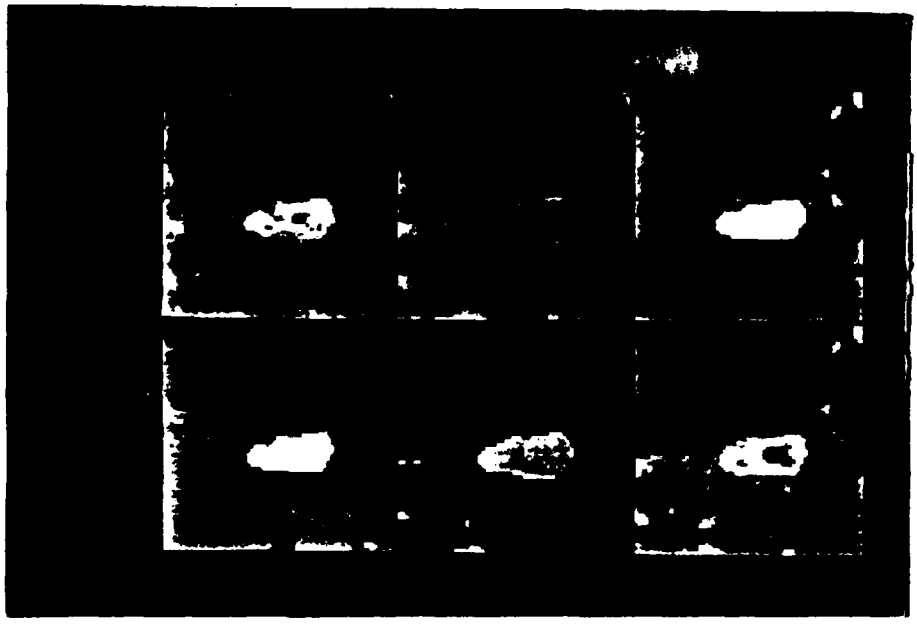

Plate 3. Portion of Record 12 of Westingnouse FLIR data tape.

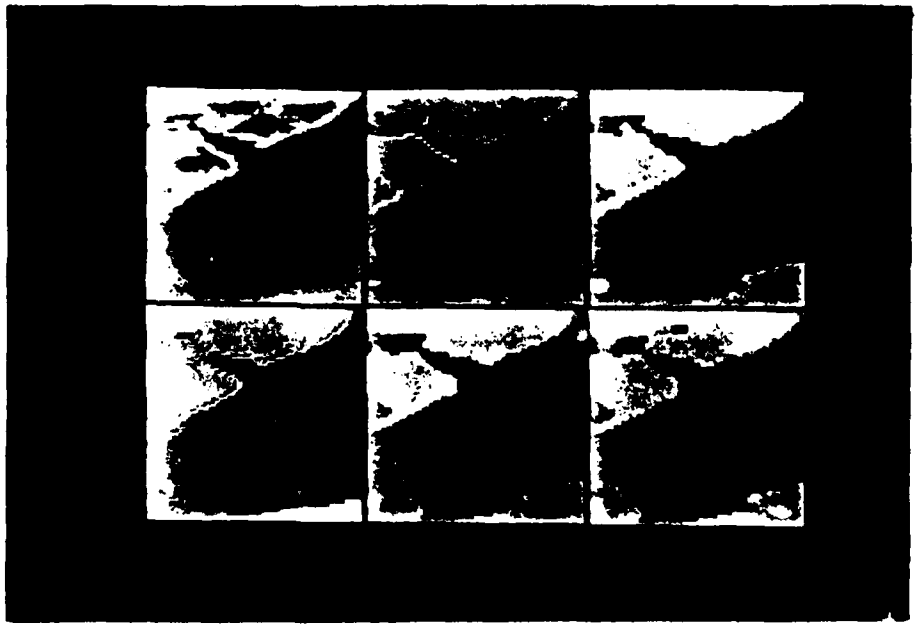

Plate 4. Portion of weather tape furnisned by Air Force Geophysical Laboratory. The picture shows the region off of the Chesapeake Bay with the edge of the Gulf Stream visible in the lower right corner.

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